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Post Optimality and Parametric Analysis with the National Bureau of Standards' Linear Programming Subroutine RVSMPX

Tyrone B. Ayers

Applied Mathematics Division

February 1975

Final

Technical Report to

Computer Services Division
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U.S. DEPARTMENT OF COMMERCE, Rogers C.B. Morton, ***Secretary***
James A. Baker, III, ***Under Secretary***
Dr. Betsy Ancker-Johnson, ***Assistant Secretary for Science and Technology***

NATIONAL BUREAU OF STANDARDS, Ernest Ambler, ***Acting Director***

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ABSTRACT

This report is a sequel to NBS Report 10695 (February 1972), The National Bureau of Standards' Linear and Quadratic Programming Subroutines, which documented one phase of an effort to provide users of the facility operated by the National Bureau of Standards' Computer Services Division, with reliable clearly-described solution algorithms for selected frequently-arising classes of special mathematical problems. The present report presents subroutines which perform post-optimality analysis and parametric programming studies on linear programming problems solved by the National Bureau of Standards' RVSMPX subroutine. (The present versions of these codes use internal storage only.)

Keywords: Algorithms; post-optimality analysis; parametric programming; linear programming.

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1. INTRODUCTION

This report documents another phase of a continuing effort to provide users of the computer facility operated by the National Bureau of Standards' Computer Services Division with reliable, clearly-described solution algorithms for selected, frequently-arising classes of mathematical problems. This phase of the effort has increased NBS's capabilities in the area of linear programming by producing three subroutines: RANGES, a subroutine which provides post-optimality ranges for the objective function coefficients (costs) and for the constants which constitute the right-hand-sides (RHS) of the linear program's constraints; PAROBJ, a subroutine which performs a parametric study of the effects of changes in the costs; and PARRHS, a subroutine which performs a similar parametric study for changes in the RHS.

The subroutines were written in FORTRAN V, solve the problems "in core," and were designed to be used with RVSMPX, a subroutine for performing the revised simplex method of linear programming. RVSMPX is described in the previous report [2], and familiarity with that document is assumed here.

Sections 2 and 3 of this report discuss, respectively, the methods used in RANGES, and those employed for PAROBJ, and PARRHS. Section 4 presents the deck set-up, and parameter settings required to use the routines, and Section 5 provides sample outputs with explanations. The appendices contain program listings, and also the results of sample problem runs made for purposes of code verification and timing.

2. SENSITIVITY ANALYSIS ON COST AND RHS VECTORS (RANGES)

The input to RANGES, consisting of a linear programming problem together with its solution, is the output from RVSMPX (see [2]). The output from RANGES is a set of upper and lower bounds. An upper-lower bound pair appears for each objective function coefficient and also for each constraint right hand side, and represents the maximum range over which that quantity can vary (with all other problem data fixed) without forcing a change from the optimal basis for the original LP problem.* That is to say, the set of variables in the optimal basis will remain the same so long as no cost or RHS is changed beyond its bounds. It should be noted that in the case of a change in an RHS, although the basis remains the same, the values of the variables in the basis would change. Correspondingly, in the case of a change in a cost, the values of the variables in the basis as well as the membership of the basis will remain the same, while the value of the objective function changes.

Note, that due to round-off error the computed bound values may occasionally vary from the actual values by as much as 3 times the epsilon (EPS) computed in RVSMPX. However, the computed values, when substituted for their corresponding cost or RHS, do indeed yield the same optimal basis.

A number of computer runs were made to check the "correctness" of the code. The times for these runs (which, by the way, employed the very same problems that were used to verify RVSMPX in [2]) are reported in Appendix D. A potentially useful statistic from these runs is that the time consumed by RANGES was on the average roughly 20% of that required by RVSMPX to solve the original problem.

---

*This technique is described in detail in [1] and [3].

3. PARAMETRIC PROGRAMMING ON COST VECTOR (PAROBJ) OR RHS VECTOR (PARRHS)

Consider the linear programming problem given in matrix notation as

$$\max cx, \text{ subject to } Ax{:}b \text{ and } x \geq 0,$$

where ":" represents $\leq$, $\geq$, or =.* If B is the basis matrix, and c_B and x_B are the corresponding subvectors of c and x respectively, then the solution to this problem is given by $x_B = B^{-1}b$, with optimal objective value of $c_B x_B$.

Parametric programming on the cost vector is concerned with replacing c in the above problem by $c^* = c + \theta f$, where f is an arbitrary vector supplied by the user and θ is a nonnegative scalar parameter. PAROBJ solves this problem by first solving the original problem using RVSMPX, then computing the largest value of θ for which the current optimal basis remains optimal. Call this value θ_{max}. A new basic solution is then calculated which is optimal in some new interval of θ values beginning with θ_{max}. The upper bound of that new interval is the new value of θ_{max}.

This procedure continues until either a.) the problem becomes unbounded (linear programming theory assures that it will remain so for all larger values of θ), or b.) the last of the critical values of θ is reached (the same solution remains optimal for all larger values of θ), or c.) a user-specified limit on θ is exceeded.

If the solution to the original problem is unbounded, it is still possible that for some positive value of θ, the solution to the modified problem "becomes" bounded. If such is the case for the original linear programming problem, then PAROBJ will start the parametric study with the smallest positive value of θ for which a bounded, feasible solution is obtained.

As with RANGES, a number of sample problems were run on PAROBJ for the purpose of code verification and timing. The results of these runs are reported in appendix D.

* * *

PARRHS is a subroutine written to perform parametric programming on the RHS vector. This is the situation in which vector b of the original problem stated in the beginning of this section is replaced by $b^* = b + \theta r$, where r is an arbitrary vector supplied by the user, and θ is as before. All the comments stated above for PAROBJ with regard to boundedness, apply for PARRHS except that the concern here is with feasibility.

---

*By way of clarification, RVSMPX accepts the A matrix in non-canonical form, so that the constraints need not be all equalities or inequalities.

4. DECK SET-UP AND PARAMETER SETTINGS

The previous sections of this report discussed the subroutines in general terms, stressing the functions they perform. This section will provide the reader with the information needed actually to use the subroutines. The deck set-ups will be discussed first, followed by the form of the call to the subroutines, followed further by an explanation of each of the parameters employed.

RANGES is used in what can be termed a "post-RVSMPX" mode. In the user's main program, after the call to RVSMPX in the form described in [2], the user simply inserts a FORTRAN CALL statement of the following form:

```
CALL RANGES (A,MA,B,MB,MT,NT,L,X,RHSUP,RHSLOW,COEUP,COELOW,NSET),
```

and the range information is printed out.

The observant reader will note that many of the parameters in the above CALL statement appear identical to those in the CALL statement for RVSMPX. In most cases they are exactly the same. There are, however, some variations, which are listed below and should be NOTED by the user.

Both PAROBJ and PARRHS operate in a "super-RVSMPX" mode. The user-supplied main program referred to earlier and discussed in [2] should pass control to one of these routines, which then takes charge of the parametric study and uses RVSMPX as a subroutine. This, of course, means that any parameters required by RVSMPX which would normally be passed directly to it, must now be passed through the parametric programming routines. The FORTRAN CALL statements, as they should appear in the main program, are:

```
CALL PAROBJ (A,MA,B,MB,MT,NT,L,X,TOLP,INV,F,BOUND,NSET),

CALL PARRHS (A,MA,B,MB,MT,NT,L,X,TOLP,INV,BOUND,R,NSET).
```

Again it should be noted that the similarity between these CALL statements and the one for RVSMPX is not accidental. A full understanding of the parameters, especially as they relate to the various subroutines, is absolutely imperative. Listed below are the parameters that appear in RANGES, PAROBJ, and PARRHS. After each variable name, there appears an (R), an (O), or an (H) if the variable is a parameter of either RANGES, PAROBJ, or PARRHS, respectively. If the variable pertains to all three subroutines without variation, no letters will appear. But if variations are present, an explanation will be given. The m and n used with reference to dimensions have the same meaning as in the RVSMPX documentation: m is the number of constraints, and n is the number of real (original) variables.

* * *

<u>A</u> (R) This is the augmented constraint matrix. It should be dimensioned at least (m+1) by (n+1) and is unchanged by the subroutine.

(O) Because of the nature of PAROBJ, A <u>must</u> be dimensioned at least (m+2) by (n+1) to provide storage for the original coefficients of the objective function. A is unchanged by PAROBJ.

(H) Because of the methods used in storing the original coefficients of the objective function and in dealing with an originally infeasible problem, A <u>must</u> be dimensioned at least (m+2) by (n+2). A is unaltered by PARRHS.

<u>MA</u> This variable equals the value of the first dimension of A. It is unchanged upon exiting from the subroutines.

<u>B</u> (R) This matrix provides storage for the inverse of the basis matrix and should be dimensioned (m+2) by (m+2). B is unchanged by the subroutine.

(O,H) Since the subroutines perform parametric studies which cover several bases, B will contain the inverse of the last optimal basis matrix upon exiting from the subroutines. B should be dimensioned at least (m+2) by (m+2).

<u>MB</u> This is the value of the first dimension of B. It is not altered by the subroutines.

<u>MT</u> (R) This is the number of rows of information in the A matrix, i.e., (m+1). MT is not changed by RANGES.

(O,H) Recall that the A matrix for PAROBJ and PARRHS has (m+2) rows. Hence MT must equal m+2. Upon exiting from the subroutine, MT is unaltered.

<u>NT</u> (R,O) This variable equals the number of columns of information in the A matrix, i.e., (n+1). It is unchanged by the subroutines.

(H) Recall that the A matrix for PARRHS has (n+2) columns. Therefore, NT must equal (n+2). Upon exiting from the subroutine, NT is not changed.

<u>L</u> (R) This is a multipurpose vector providing work space needed by RVSMPX. L is not changed in any way by RANGES.

(O,H) In order to maintain a certain consistency in output formulation, the user-supplied values of L only function for the initial reporting of the problem, after which certain elements of L are automatically set by the subroutines. Upon exiting, L(5), L(8), L(13), and L(14) will equal 1. L(4) and L(12) will equal 0. The value of all other L(I), I $\leq$ 14, will be the same as they are upon returning from RVSMPX.

<u>X</u> (R) Just as in RVSMPX, X provides storage for the solution of the problem. X is not changed by RANGES.

(O,H) These subroutines perform a parametric study over a series of optimal bases. Therefore, the solution in X upon exiting from the subroutines will not be the original solution, but rather the solution corresponding to the final optimal basis of the parametric study.

<u>TOLP</u> (O,H) This is the variable used by RVSMPX to compute the value of EPS and CEPS. Its value on input should be exactly the same as it is for RVSMPX. TOLP is not changed by the subroutine.

INV (O,H) This is the switch allowing the user to supply RVSMPX with an initial feasible solution. As in RVSMPX, INV may still be used to indicate a user-supplied initial basis. The value of INV is altered in the subroutines.

RHSUP (R) This vector provides storage for the upper bounds on the ranges of the right-hand sides of the given problem being solved. Since each right-hand side will have an upper bound, RHSUP should be dimensioned at least m in the main program, where m is the number of rows in the unaugmented A matrix.

RHSLOW (R) This vector provides storage for the lower bounds on the ranges of the right-hand sides. Like RHSUP, RHSLOW should be dimensioned at least m in the main program.

COEUP (R) This vector provides storage for the upper bounds on the ranges of the coefficients of the objective function of the given problem. COEUP should be dimensioned at least n in the main program, where n is the number of columns in the unaugmented A matrix.

COELOW (R) This vector provides storage for the lower bounds on the ranges of the coefficients. Like COEUP, COELOW should be dimensioned at least n.

R (H) This is the vector supplied by the user to indicate the rates of change of each of the right-hand sides with respect to the scalar θ. Therefore, R must be dimensioned at least m. R should be set up so that the ith element of R is the rate of change of the ith right-hand side. For example, if the original right-hand side vector is $b=(1,2,3.7,5)^T$ and if R is $(0,2,-3,.6)^T$, then the new right hand side vector b^* varies as follows:

$$b_1^* = b_1 + 0\,\theta = 1$$

$$b_2^* = b_2 + 2\,\theta = 2 + 2\,\theta$$

$$b_3^* = b_3 - 3\,\theta = 3.7 - 3\,\theta$$

$$b_4^* = b_4 + 0.6\,\theta = 5 + 0.6\theta$$

where $\theta \geq 0$. The subroutine does not change R.

F (O) This is the vector supplied by the user to indicate the rates of change of each of the objective function coefficients with respect to the scalar θ. Therefore, F must be dimensioned by at least n. F should be set up so that the jth element of F is the rate of change of the jth cost coefficient. For example, if the original cost vector is $c = (2,3,1.6,-4)$ and if F is $(2,0,-.5,5)$, then the new cost vector c^* varies as follows:

$$c_1^* = c_1 + 2\theta = 2 + 2\theta$$

$$c_2^* = c_2 + 0\theta = 3$$

$$c_3^* = c_3 - 0.5\theta = 1.6 - 0.5\theta$$

$$c_4^* = c_4 + 5\theta = -4 + 5\theta$$

where $\theta \geq 0$. F is not altered by the subroutine.

BOUND (O,H) This is the user-specified limit on the non-negative scalar parameter θ. The scalar θ is incremented up to the value of BOUND. Once the value of θ exceeds BOUND, the subroutines will terminate by reporting that the limit has been exceeded and that the current basis remains optimal at that limit. However, the subroutines may terminate before θ reaches BOUND if:

1) the largest critical value of θ is less than BOUND,
2) the problem becomes infeasible (PARRHS),
3) the problem becomes unbounded (PAROBJ).

The value of BOUND is unchanged by the subroutines.

NSET (R) This parameter is a user-specified switch. If the user sets NSET = 0, ranges are calculated for both right-hand sides and objective function coefficients. If NSET = 1, ranges are determined only for the right-hand sides. If NSET = 2, ranges are obtained only for the objective function coefficients.

(O,H) For these subroutines, this user-specified switch determines the output format. If NSET = 0, a detailed format is used. If NSET = 1, a compressed tabular summary format is used.

5. OUTPUT

This section of the report will discuss the output from the three subroutines, and illustrate them through actual output samples.

5.1 THE OUTPUT FROM RANGES

The printing of the output is controlled by RANGES. Information on the original right-hand side vector is printed first, followed by information on the objective function coefficient vector. The user may obtain information on both the right-hand side and the objective function coefficient vectors or either of them separately, depending on his specified value for the parameter NSET (see section 4). The output is printed in an E14.8 format as follows:

```
-.14845019+02  LE   RHS  1   LE  POS.INFINITY
 .54586695+01  LE   RHS  2   LE  POS.INFINITY
 .30275573+01  LE   RHS  3   LE  .15395222+02
 .15161557+02  LE   RHS  4   LE  POS.INFINITY

 NEG.INFINITY  LE   COEFFICIENT  1   LE -.14055693+00
-.20215248+01  LE   COEFFICIENT  2   LE -.64413020+00
 NEG.INFINITY  LE   COEFFICIENT  3   LE -.54610899+01
 NEG.INFINITY  LE   COEFFICIENT  4   LE -.40288613+01
-.12985771-01  LE   COEFFICIENT  5   LE  .57053266+00
```

Line 1, for example, reads -14.845019 $\leq$ RHS 1 $\leq$ POSITIVE INFINITY. This means that the reported final basis will remain optimal as long as RHS 1 (the first element of the right-hand side vector of the problem being solved) is greater than -14.85019. Similarly, bounds are reported for each remaining element of the right-hand side vector, i.e., RHS 2, RHS 3, and RHS 4.

Line 5 reads

NEGATIVE INFINITY $\leq$ COEFFICIENT 1 $\leq$ -.14055693.

This means that coefficient 1, the first element of the cost vector of the problem being solved, has no lower bound and has an upper bound of -.14055693. Following line 5 are the bounds on the remaining elements of the cost vector.

5.2 THE OUTPUT FROM PAROBJ AND PARRHS

All output is completely controlled by the subroutines, and the output formats for the two routines are nearly identical. The first item to be printed is a statement that the first critical value of ϵ is zero. With this value of ϵ, the solution to the parametric programming problem is the same as the solution to the simple linear programming problem, and that

solution is printed next. A formula for the calculation of the objective function value for all values of θ from zero through the next critical value is then printed.

A cycle of output then begins. The "current" critical value of θ is printed, along with the basis change required when passing through that value for θ. The "current" basis is then given, followed by the objective function calculation formula. This output grouping is repeated until one of the termination conditions in section 3 is met. This output format is illustrated in Figures 1 and 2.

A more compressed tabular format is also available. The value of NSET (see Section 4) controls the output format selection. The compressed format is illustrated in Figures 3 and 4.

```
CRITICAL VALUE (   0) OF THETA =    .00000000

EPSILON =    .364000-04  CAPITAL EPSILON =    .364000-03      0 NON-ZERO ENTRIES ARE EFFECTIVELY EQUAL TO ZERO.

END OF PHASE 1.    OBJECTIVE FUNCTION =      .74505806-08    THERE WERE   2 ITERATIONS.
MINIMUM PIVOT WAS   1.0000    AT ITERATION   1.   REAL OBJECTIVE FUNCTION =  -1.2676858

END OF PHASE 2.    OBJECTIVE FUNCTION =     -.66359763       THERE WERE   1 ITERATIONS.
MINIMUM PIVOT WAS   1.0000    AT ITERATION   1.

BASIC.VARIABLES

   X(  1)=   .2211992

   SI  1)=   6.538539      S(  2)=   10.48937      S(  3)=   12.37799      S(  5)=   .1006814

OBJECTIVE FUNCTION VALUE BETWEEN CRITICAL VALUES (  0) AND (  1) OBTAINED FROM   .44239841+00+ THETA   -.66359762+00

CRITICAL VALUE (  1) OF THETA =   .62500000+00
AT THAT VALUE, X(  4) ENTERS THE BASIS AND SI   1) LEAVES.
```

Figure 1: PAROBJ Output
Regular Format

```
BASIC VARIABLES

   X(  1)=    .5184055       X(  4)=    2.080444

   S(  2)=   12.56981        S(  3)=    9.405929       S(  5)=    2.478332

OBJECTIVE FUNCTION VALUE = -.38709863+00
OBJECTIVE FUNCTION VALUE BETWEEN CRITICAL VALUES (  1) AND (  2) OBTAINED FROM    .51976991+01* THETA    -.36356606+01

 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

CRITICAL VALUE (  2) OF THETA =   .86764706+00
AT THAT VALUE, X(   5) ENTERS THE BASIS AND X(   1) LEAVES.

BASIC VARIABLES

   X(  4)=    5.579681       X(  5)=    1.425615

   S(  2)=   27.47397        S(  3)=    .3338330       S(  5)=    9.736009

OBJECTIVE FUNCTION VALUE =   .87410781+00
OBJECTIVE FUNCTION VALUE BETWEEN CRITICAL VALUES (  2) AND (  3) OBTAINED FROM    .14010593+02* THETA    -.11282142+02

 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

CRITICAL VALUE (  3) OF THETA =   .12500000+01
AT THAT VALUE, S(   4) ENTERS THE BASIS AND S(   3) LEAVES.

BASIC VARIABLES

   X(  4)=    5.746590       X(  5)=    1.592532

   S(  2)=   28.30855        S(  4)=    .6676660       S(  5)=   10.40367

OBJECTIVE FUNCTION VALUE =   .62310992+01
OBJECTIVE FUNCTION VALUE BETWEEN CRITICAL VALUES (  3) AND (  4) OBTAINED FROM    .14678259+02* THETA    -.12116724+02

 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

CRITICAL VALUE (  4) OF THETA =   .16111111+01
PROBLEM GOES UNBOUNDED BEYOND THIS CRITICAL VALUE.
```

Figure 1: PAROBJ Output
Regular Format (continued)

```
CRITICAL VALUE ( 0) OF THETA =  .00000000

EPSILON =  .364000-04  CAPITAL EPSILON =  .364000-03   0 NON-ZERO ENTRIES ARE EFFECTIVELY EQUAL TO ZERO.

**********************************************************************************************
*                                                                                            *
*      END OF PHASE 1.   OBJECTIVE FUNCTION =  .74505806-08    THERE WERE  2 ITERATIONS.     *
*      MINIMUM PIVOT WAS   1.0000   AT ITERATION   1.   REAL OBJECTIVE FUNCTION =  -1.2676858 *
*                                                                                            *
**********************************************************************************************

**********************************************************************************************
*                                                                                            *
*      END OF PHASE 2.    OBJECTIVE FUNCTION =    -.66359763       THERE WERE   1 ITERATIONS.  *
*      MINIMUM PIVOT WAS   1.0000   AT ITERATION   1.                                        *
*                                                                                            *
**********************************************************************************************
BASIC VARIABLES

  X(  1)=   .2211992

  S(  1)=   6.538539     S(  2)=   10.48937     S(  3)=   12.37799     S(  5)=   .1006814

OBJECTIVE FUNCTION VALUE BETWEEN CRITICAL VALUES (  0) AND (  1) OF THETA OBTAINED FROM  =.85714285+00*THETA   =.66359763+00

 *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *

CRITICAL VALUE (  1) OF THETA =   .58730794-01
AT THAT VALUE, X(  5) ENTERS THE BASIS AND S(  5) LEAVES.

BASIC VARIABLES

  X(  1)=   .2379794     X(  5)=   .0000000

  S(  1)=   6.521758     S(  2)=   10.43064     S(  3)=   12.68004
```

Figure 2: PARRHS Output
Regular Format

```
OBJECTIVE FUNCTION VALUE = -.71393830+00
OBJECTIVE FUNCTION VALUE BETWEEN CRITICAL VALUES (  1) AND (  2) OF THETA OBTAINED FROM  -.22500000+01*THETA    -.58179401+00

 .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .

CRITICAL VALUE (  2) OF THETA =   .10106485+01
AT THAT VALUE, X(   4) ENTERS THE BASIS AND X(   1) LEAVES.

BASIC VARIABLES

   X(  4)=   .0000000       X(  5)=   .7139383

   S(  1)=   11.75731       S(  2)=   15.19023       S(  3)=   15.53579

OBJECTIVE FUNCTION VALUE = -.28557532+01
OBJECTIVE FUNCTION VALUE BETWEEN CRITICAL VALUES (  2) AND (  3) OF THETA OBTAINED FROM  -.25000000+01*THETA    -.32913187+00

 .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .

THE CRITICAL VALUE OF THETA IS INFINITE.

 .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
```

Figure 2: PARRHS Output
Regular Format (continued)

```
THE R VECTOR IS AS FOLLOWS
R(   1) =  2.00
R(   2) =  1.00
R(   3) =  4.00
R(   4) =  2.00
R(   5) =  2.00

BOUND=  6.00


CRITICAL VALUE (  0) OF THETA =   .00000000


EPSILON =    .364000-04  CAPITAL EPSILON =    .364000-03      0 NON-ZERO ENTRIES ARE EFFECTIVELY EQUAL TO ZERO.


********************************************************************************************************
*                                                                                                      *
*          END OF PHASE 1.    OBJECTIVE FUNCTION =      .74505806-08    THERE WERE   2 ITERATIONS.     *
*          MINIMUM PIVOT WAS   1.0000    AT ITERATION   1.   REAL OBJECTIVE FUNCTION =  -1.2676858     *
*                                                                                                      *
********************************************************************************************************


********************************************************************************************************
*                                                                                                      *
*          END OF PHASE 2.    OBJECTIVE FUNCTION =     -.66359763       THERE WERE   1 ITERATIONS.     *
*          MINIMUM PIVOT WAS   1.0000    AT ITERATION   1.                                             *
*                                                                                                      *
********************************************************************************************************
BASIC VARIABLES

  X(  1)=     .221199

  S(  1)=    6.538539    S(  2)=   10.489368    S(  3)=   12.377992    S(  5)=     .100681

OBJECTIVE FUNCTION VALUE BETWEEN CRITICAL VALUES (  0) AND (  1) OF THETA OBTAINED FROM  -.85714285+00*THETA    -.66359763+00

 *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
```

Figure 3: PAROBJ Output
Compressed Format

```
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

CRIT VAL     CRIT VAL     BASIS INFORMATION      OBJ FUNC VAL     OBJ FUNC VAL BETWEEN THIS
  NO.                     ENTERS     LEAVES                           AND NEXT CRIT VAL

   1           .6250      X(  4)     S(  1)         -.3871          5.1977* THETA    -3.6357

   2           .8676      X(  5)     X(  1)          .8741         14.0106* THETA   -11.2821

   3          1.2500      S(  4)     S(  3)         6.2311         14.6783* THETA   -12.1167

   4          1.6111      NO FURTHER INFORMATION-PROBLEM GOES UNBOUNDED BEYOND THIS CRITICAL VALUE
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
```

Figure 3: PAROBJ Output
Compressed Format (continued)

```
THF F VECTOR IS AS FOLLOWS
F(   1) =  2.00
F(   2) =  1.00
F(   3) =  4.00
F(   4) =  2.00
F(   5) =  2.00
BOUND=  6.00


CRITICAL VALUE (  0) OF THETA =   .00000000


EPSILON =    .364000-04  CAPITAL EPSILON =    .364000-03      0 NON-ZERO ENTRIES ARE EFFECTIVELY EQUAL TO ZERO.


*******************************************************************************************************
*                                                                                                     *
*           END OF PHASE 1.    OBJECTIVE FUNCTION =      .74505806-08    THERE WERE   2 ITERATIONS.   *
*           MINIMUM PIVOT WAS   1.0000    AT ITERATION   1.   REAL OBJECTIVE FUNCTION =  -1.2676858   *
*                                                                                                     *
*******************************************************************************************************


*******************************************************************************************************
*                                                                                                     *
*           END OF PHASE 2.    OBJECTIVE FUNCTION =     -.66359763       THERE WERE   1 ITERATIONS.   *
*           MINIMUM PIVOT WAS   1.0000    AT ITERATION   1.                                           *
*                                                                                                     *
*******************************************************************************************************
BASIC VARIABLES

  X(  1)=     .221199

  S(  1)=    6.538539    S(  2)=   10.489368    S(  3)=   12.377992    S(  5)=     .100681

OBJECTIVE FUNCTION VALUE BETWEEN CRITICAL VALUES (  0) AND (  1) OBTAINED FROM    .44239841+00* THETA    -.66359762+00
```

Figure 4: PARRHS Output
Compressed Format

CRIT VAL NO.	CRIT VAL	BASIS INFORMATION ENTERS	LEAVES	OBJ FUNC VAL	OBJ FUNC VAL BETWEEN THIS AND NEXT CRIT VAL	
1	.0587	X(5)	S(5)	-.7139	-2.2500* THETA	-.5818
2	1.0106	X(4)	X(1)	-2.8558	-2.5000* THETA	-.3291

CRITICAL VALUE (3) OF THETA IS INFINITE.

* *

Figure 4: PARRHS Output
Compressed Format (continued)

6. REFERENCES

1. Hadley, G., Linear Programming, Addison-Wesley, Reading, Mass., 1962.

2. Hall, W.G., Jackson, R.H.F., Saunders, P.B., The National Bureau of Standards' Linear and Quadratic Programming Subroutines, NBS Report No. 10695, February 1972.

3. Simonnard, M., Linear Programming, Prentice Hall, Englewood Cliffs, N.J., 1966.

APPENDIX A: Listing of RANGES

```
WFOR,IS RANGES
FOR 010A-09/21/73-21:06:09 (,0)

   SUBROUTINE RANGES     ENTRY POINT 001274

   STORAGE USED: CODE(1) 001344; DATA(0) 000234; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003   NWDU$
    0004   N102$
    0005   NEXP1$
    0006   NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001   000331 1L       0001   000453 106L     0000   000033 11F      0001   000120 12L      0001   000473 120L
    0000   000042 13F      0001   000534 130L     0000   000053 14F      0001   000166 144G     0000   000064 15F
    0001   000555 150L     0001   000174 151G     0001   000312 16L      0001   000227 167G     0001   000322 17L
    0001   000605 170L     0001   000351 18L      0001   000654 180L     0001   000665 190L     0001   000214 2L
    0001   000714 200L     0001   000712 201L     0001   000726 220L     0001   000771 230L     0001   000416 234G
    0001   000424 237G     0001   000447 250G     0001   001012 250L     0001   000470 261G     0001   000510 267G
    0001   001042 270L     0001   001122 280L     0001   001133 290L     0001   001161 300L     0001   001157 301L
    0001   000550 305G     0001   001216 310L     0000   000100 311F     0001   000577 315G     0001   001226 320L
    0000   000112 321F     0001   001243 322L     0001   001235 330L     0000   000124 331F     0001   000723 352G
    0001   000750 360G     0001   001035 375G     0001   001034 405G     0000   000135 500F     0001   000247 7L
    0001   000037 8L       0001   000061 9L       0000 1 000016 I        0000   000145 INJP$    0000 1 000011 J
    0000 1 000023 JTEL     0000 1 000022 JI       0000 1 000024 J2       0000 1 000025 J3       0000 1 000030 J4
    0000 1 000010 K        0000 1 000020 KL       0000 1 000014 KU       0000 1 000006 L1       0000 1 000005 L1OFF
    0000 1 000007 L2       0000 1 000013 L2OFF    0000 1 000000 M        0000 1 000003 MP1      0000 1 000004 MP2
    0000 1 000021 M2       0000 1 000001 N        0000 1 000032 NDWN     0000 1 000012 NPLPG    0000 1 000002 NP1
    0000 1 000031 NUPP     0000 R 000017 RATIO    0000 R 000015 RATMIN   0000 R 000026 XX       0000 R 000027 YY


00101      1*          SUBROUTINE RANGES(A,MA,B,MB,MT,NT,L,X,RHSUP,RHSLOW,COEUP,COELOW,NS  RG1000
00101      2*         1ET)                                                                 RG1005
00103      3*          DIMENSION A(MA,1),B(MB,1),L(1),X(1),RHSUP(1),RHSLOW(1)              RG1010
00104      4*          DIMENSION COEUP(1),COELOW(1)                                        RG1020
00105      5*          IF(L(3).EQ.0.OR.L(3).EQ.1) GO TO 8                                  RG1030
00107      6*          WRITE(6,15)                                                         RG1040
00111      7*          GO TO 322                                                           RG1050
00112      8*        8 M=MT-1                                                              RG1060
00113      9*          N=NT-1                                                              RG1070
00114     10*          NP1=N+1                                                             RG1080
00115     11*          MP1=M+1                                                             RG1090
00116     12*          MP2=M+2                                                             RG1100
00117     13*          L1OFF=14                                                            RG1110
00120     14*          L1=0                                                                RG1120
00121     15*          L2=0                                                                RG1130
00122     16*          K=NP1                                                               RG1140
00123     17*        9 J=L(L1OFF+K)                                                        RG1150
00124     18*          IF(J.EQ.3) GO TO 12                                                 RG1160
```

```
00126          IF(J.EQ.1) L1=L1+1                                            RG1170
00130          IF(J.EQ.2) L2=L2+1                                            RG1180
00132          IF((L1+L2).EQ.M) GO TO 12                                     RG1190
00134          K=K+1                                                         RG1200
00135          GO TO 9                                                       RG1210
00136       12 NPLPG=M+L1+L2                                                 RG1220
00137          L2OFF=L1OFF+M+N+L2                                            RG1230
00140          NPLPG=M+L1+L2                                                 RG1240
00141          IF(NSET.EQ.2) GO TO 18                                        RG1245
00141    C ***************************************************              RG1250
00141    C **BEGIN RHS UPPER BOUND COMPUTATION PROCEEDURE**                 RG1260
00141    C ***************************************************              RG1270
00143          DO 1 J=1,M                                                    RG1280
00146          KU=0                                                          RG1290
00147          RATMIN=1E35                                                   RG1300
00147    C **CHECK FOR NEGATIVE VALUES IN INVERSE OF BASIS MATRIX**         RG1310
00150          DO 2 I=1,M                                                    RG1320
00153          IF (B(I,J).GE.0.0) GO TO 2                                    RG1330
00155          KU=1                                                          RG1340
00155    C **RATIO OF OPTIMAL BASIC SOLUTION TO INVERSE OF BASIS MATRIX**   RG1350
00156          RATIO=-B(I,MP2)/B(I,J)                                        RG1360
00156    C **FIND MINIMUM RATIO**                                            RG1370
00157          IF(RATIO.LT.RATMIN) RATMIN=RATIO                              RG1380
00161        2 CONTINUE                                                      RG1390
00161    C **COMPUTE UPPER BOUND ON RIGHT HAND SIDE**                        RG1400
00163          RHSUP(J)=RATMIN+A(J,NP1)                                      RG1410
00163    C ***************************************************              RG1420
00163    C **BEGIN RHS LOWER BOUND COMPUTATION PROCEEDURE**                 RG1430
00163    C ***************************************************              RG1440
00164        6 KL=0                                                          RG1450
00165          RATMIN=1E35                                                   RG1460
00165    C **CHECK FOR POSITIVE VALUES IN INVERSE OF BASIS MATRIX**         RG1470
00166          DO 7 I=1,M                                                    RG1480
00171          IF (B(I,J).LE.0.0) GO TO 7                                    RG1490
00173          KL=1                                                          RG1500
00173    C **RATIO OF OPTIMAL BASIC SOLUTION TO INVERSE OF BASIS MATRIX**   RG1510
00174          RATIO=B(I,MP2)/B(I,J)                                         RG1520
00174    C **FIND MINIMUM RATIO**                                            RG1530
00175          IF(RATIO.LT.RATMIN) RATMIN=RATIO                              RG1540
00177        7 CONTINUE                                                      RG1550
00177    C **COMPUTE LOWER BOUND ON RIGHT HAND SIDE**                        RG1560
00201          RHSLOW(J)=A(J,NP1)-RATMIN                                     RG1570
00201    C *********************************                                 RG1580
00201    C **BEGIN PRINTING OF RHS BOUNDS**                                  RG1590
00201    C *********************************                                 RG1600
00202       10 IF(KL.EQ.0.AND.KU.GT.0) GO TO 16                              RG1610
00204          IF(KL.GT.0.AND.KU.EQ.0) GO TO 17                              RG1620
00206          WRITE(6,11)RHSLOW(J),J,RHSUP(J)                               RG1630
00213          GO TO 1                                                       RG1640
00214       16 WRITE(6,13)J,RHSUP(J)                                         RG1650
00220          GO TO 1                                                       RG1660
00221       17 WRITE(6,14)RHSLOW(J),J                                        RG1670
00225        1 CONTINUE                                                      RG1680
00227          WRITE(6,500)                                                  RG1690
00231          IF(NSET.EQ.1) GO TO 322                                       RG1695
00231    C ***************************************************************** RG1700
00231    C **BEGIN OBJECTIVE FUNCTION UPPER BOUND COMPUTATION PROCEDURE      RG1710
00231    C ***************************************************************** RG1720
```

```
00233          18 DO 330 J=1,N
00236             DO 100 I=1,M
00241         100 B(MP2,I)=0.0
00243             M2=0
00243       C ****************************************************
00243       C **STORE APPROPRIATE ROW OF B INVERSE IN ROW M+2 OF B**
00243       C ****************************************************
00244             IF(X(J).LT.0.0) GO TO 120
00246             M2=1
00247             DO 105 J1=1,M
00252             K=L(L2OFF+J1)
00253             JTEL=J1
00254             IF(K.EQ.J) GO TO 106
00256         105 CONTINUE
00260         106 DO 110 J2=1,M
00263         110 B(MP2,J2)=B(JTEL,J2)
00263       C **COMPUTE INDICATORS AND DENOMINATORS**
00265         120 RATMIN=1E35
00266             DO 190 J2=1,NPLPG
00271             IF(X(J2).GE.0.0) GO TO 190
00273             J3=L(L1OFF+J2)
00274             IF(J3.GT.0) GO TO 170
00274       C **REAL VARIABLES**
00276             XX=0.0
00277             IF(J2.EQ.J) XX=-1.0
00301             IF(M2.EQ.1) GO TO 130
00303             GO TO 150
00304         130 DO 140 I=1,M
00307         140 XX=XX+B(MP2,I)*A(I,J2)
00311         150 IF(XX.GE.0.0) GO TO 190
00313             YY=A(MP1,J2)
00314             DO 160 I=1,M
00317         160 YY=YY+B(MP1,I)*A(I,J2)
00321             GO TO 180
00322         170 J4=J2-N
00322       C **SLACK AND SURPLUS VARIABLES**
00323             XX=0.0
00324             IF(M2.EQ.1) XX=(-1)**(J3+1)*B(MP2,J4)
00326             IF(XX.GE.0.0) GO TO 190
00330             YY=(-1)**(J3+1)*B(MP1,J4)
00330       C **COMPUTE RATIO OF INDICATORS TO DENOMINATORS**
00331         180 RATIO=YY/XX
00331       C **FIND MINIMUM RATIO**
00332             IF(RATIO.GE.RATMIN) GO TO 190
00334             RATMIN=RATIO
00335         190 CONTINUE
00337             NUPP=0
00340             IF(RATMIN.GT.1E34) NUPP=1
00342             IF(NUPP.EQ.1) GO TO 201
00342       C **COMPUTE UPPER BOUND ON COEFFICIENT**
00344             COEUP(J)=RATMIN+A(MP1,J)
00345             GO TO 200
00346         201 COEUP(J)=RATMIN
00346       C ************************************************************
00346       C **BEGIN OBJECTIVE FUNCTION LOWER BOUND COMPUTATION PROCEDURE
00346       C ************************************************************
00347         200 IF(M2.EQ.0) GO TO 220
00347       C **CHANGE SIGN OF ROW M+2 OF B**
```

```
00351            DO 210 J1=1,M                                                RG2310
00354        210 B(MP2,J1)=-B(MP2,J1)                                         RG2320
00354      C **COMPUTE INDICATORS AND DENOMINATORS**                          RG2330
00356        220 RATMIN=1E35                                                  RG2340
00357            DO 290 J2=1,NPLPG                                            RG2350
00362            IF(X(J2).GE.0.0) GO TO 290                                   RG2360
00364            IF(J3.GT.0) GO TO 270                                        RG2370
00364      C **REAL VARIABLES**                                               RG2380
00366            XX=0.0                                                       RG2390
00367            IF(J2.EQ.J) XX=1.0                                           RG2400
00371            IF(M2.EQ.1) GO TO 230                                        RG2410
00373            GO TO 250                                                    RG2420
00374        230 DO 240 I=1,M                                                 RG2430
00377        240 XX=XX+B(MP2,I)*A(I,J2)                                       RG2440
00401        250 IF(XX.GE.0.0) GO TO 290                                      RG2450
00403            YY=A(MP1,J2)                                                 RG2460
00404            DO 260 I=1,M                                                 RG2470
00407        260 YY=YY+B(MP1,I)*A(I,J2)                                       RG2480
00411            GO TO 280                                                    RG2490
00412        270 J4=J2-N                                                      RG2500
00412      C **SLACK AND SURPLUS VARIABLES**                                  RG2510
00413            XX=0.0                                                       RG2520
00414            IF(M2.EQ.1) XX=(-1)**(J3+1)*B(MP2,J4)                        RG2530
00416            IF(XX.GE.0.0) GO TO 290                                      RG2540
00420            YY=(-1)**(J3+1)*B(MP1,J4)                                    RG2550
00420      C **COMPUTE RATIO OF INDICATORS TO DENOMINATORS**                  RG2560
00421        280 RATIO=YY/XX                                                  RG2570
00421      C **FIND MINIMUM RATIO**                                           RG2580
00422            IF(RATIO.GE.RATMIN) GO TO 290                                RG2590
00424            RATMIN=RATIO                                                 RG2600
00425        290 CONTINUE                                                     RG2610
00427            NDWN=0                                                       RG2620
00430            IF(RATMIN.GT.1E34) NDWN=1                                    RG2630
00432            IF(NDWN.EQ.1) GO TO 301                                      RG2640
00432      C **COMPUTE LOWER BOUND ON COEFFICIENT**                           RG2650
00434            COELOW(J)=-RATMIN+A(MP1,J)                                   RG2660
00435            GO TO 300                                                    RG2670
00436        301 COELOW(J)=-RATMIN                                            RG2680
00436      C ********************************************                     RG2690
00436      C **BEGIN PRINTING OF OBJECTIVE FUNCTION BOUNDS**                  RG2700
00436      C ********************************************                     RG2710
00437        300 IF(NUPP.EQ.0.AND.NDWN.EQ.1) GO TO 310                        RG2720
00441            IF(NUPP.EQ.1.AND.NDWN.EQ.0) GO TO 320                        RG2730
00443            WRITE(6,331) COELOW(J),J,COEUP(J)                            RG2740
00450            GO TO 330                                                    RG2750
00451        310 WRITE(6,311) J,COEUP(J)                                      RG2760
00455            GO TO 330                                                    RG2770
00456        320 WRITE(6,321) COELOW(J),J                                     RG2780
00462        330 CONTINUE                                                     RG2790
00464         11 FORMAT(E14.8,2X,'LE   RHS',I3,'   LE',E14.8)                 RG2800
00465         13 FORMAT('  NEG.INFINITY  LE   RHS',I3,'   LE',E14.8)          RG2810
00466         14 FORMAT(E14.8,2X,'LE   RHS',I3,'   LE  POS.INFINITY')         RG2820
00467         15 FORMAT('  NO RANGING DONE DUE TO EITHER INFEASIBILITY OR UNBOUNDEDN RG2830
00467           1ESS.')                                                       RG2840
00470        311 FORMAT('  NEG.INFINITY  LE   COEFFICIENT',I3,'   LE',E14.8)  RG2850
00471        321 FORMAT(E14.8,2X,'LE   COEFFICIENT',I3,'   LE  POS.INFINITY') RG2860
00472        331 FORMAT(E14.8,2X,'LE   COEFFICIENT',I3,'   LE',E14.8)         RG2870
00473        500 FORMAT(/////)                                                RG2880
```

```
00474    193*       322 RETURN                                          RG2890
00475    194*           END                                             RG2900

      END OF COMPILATION:        NO  DIAGNOSTICS.

@ FIN
```

APPENDIX B: Listing of PAROBJ

```
@FOR,IS PAROBJ
FOR 010A-09/20/73-11:43:01 (,0)

  SUBROUTINE PAROBJ    ENTRY POINT 002743

  STORAGE USED: CODE(1) 003156; DATA(0) 000533; BLANK COMMON(2) 000000

  EXTERNAL REFERENCES (BLOCK, NAME)

   0003   RVSMPX
   0004   NWDU$
   0005   NI02$
   0006   NI01$
   0007   NEXP1$
   0010   NERR3$

  STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

   0000   000126 10F      0001   000415 100L     0001   001736 1001L    0001   002021 1002L    0001   002073 1003L
   0000   000277 1004F    0000   000320 1005F    0000   000337 1006F    0000   000354 1007F    0000   000362 1009F
   0000   000366 1010F    0001   002411 1021G    0001   002323 1099L    0001   000157 11L      0001   002335 1100L
   0001   002617 1101G    0001   002346 1101L    0000   000371 1102F    0000   000406 1103F    0000   000413 1104F
   0000   000420 1105F    0001   002660 1123G    0001   002707 1130G    0001   000532 130L     0001   000130 143G
   0001   000131 146G     0001   000203 160G     0001   000217 167G     0000   000127 201F     0001   000335 226G
   0001   000670 240L     0001   000411 254G     0001   000753 260L     0001   000470 261G     0001   000476 267G
   0001   000764 270L     0001   000525 275G     0001   001016 300L     0001   000551 307G     0001   000602 320G
   0001   000623 327G     0001   001134 330L     0001   000626 333G     0001   001202 340L     0001   000657 345G
   0001   001221 350L     0001   000733 361G     0000   000051 4F       0001   001250 400L     0000   000143 401F
   0000   000157 402F     0001   001011 407G     0001   001054 416G     0001   001105 427G     0001   001126 437G
   0000   000062 5F       0001   001340 500L     0001   001333 520G     0001   001361 526G     0001   001457 543G
   0001   001463 547G     0001   001541 557G     0001   001573 566G     0000   000063 6F       0001   001422 600L
   0001   001703 617G     0001   001477 620L     0000   000064 7F       0001   001545 700L     0001   002114 706G
   0001   001614 710L     0001   001755 720L     0001   001762 730L     0001   001772 740L     0001   002004 750L
   0000   000204 751F     0001   002130 760L     0001   002277 762G     0001   000351 8L       0001   002304 800L
   0001   002425 810L     0000   000101 9F       0001   002624 900L     0000   000220 901F     0000   000230 902F
   0000   000241 903F     0000   000254 904F     0000   000266 905F     0000   000270 906F     0001   002632 910L
   0001   002640 920L     0001   002646 930L     0001   002664 999L     0000 R 000016 CRIT     0000 R 000040 DENOM
   0000 R 000037 ENUM     0000 R 000021 EPS      0000 I 000024 I        0000   000456 INJP$    0000 I 000030 ISG
   0000 I 000025 J        0000 I 000033 J1       0000 I 000035 J2       0000 I 000044 J5       0000 I 000017 KT
   0000 I 000020 KT1      0000 I 000032 K1       0000 I 000042 K3       0000 I 000046 LEAVE    0000 I 000047 LET1
   0000 I 000050 LET2     0000 I 000006 L1       0000 I 000014 L1OFF    0000 I 000007 L2       0000 I 000015 L2OFF
   0000 I 000010 L3       0000 I 000001 M        0000 I 000002 MP1      0000 I 000003 MP2      0000 I 000031 M2
   0000 I 000000 N        0000 I 000011 NPL      0000 I 000013 NPLPG    0000 I 000012 NPLP1    0000 I 000004 NP1
   0000 I 000041 NTER     0000 R 000026 R        0000 R 000036 RATIO    0000 R 000034 RATMIN   0000 R 000027 S
   0000 R 000043 SMALL    0000 R 000005 STAR     0000 R 000045 TEST     0000 R 000022 XX       0000 R 000023 YY


00101      1*          SUBROUTINE PAROBJ(A,MA,B,MB,MT,NT,L,X,TOLP,INV,F,BOUND,NSET)      OB1000
00103      2*          DIMENSION A(MA,1),B(MB,1),L(1),X(1),F(1)                         OB1010
00104      3*          N=NT-1                                                           OB1020
00105      4*          M=MT-2                                                           OB1030
00106      5*          MP1=M+1                                                          OB1040
```

```
00107      MP2=MP1+1                                          OB1050
00110      NP1=NT                                             OB1060
00111      MT=MP1                                             OB1070
00112      STAR='  *'                                         OB1071
00113      L1=L(1)                                            OB1080
00114      L2=L(2)                                            OB1090
00115      L3=L(3)                                            OB1100
00116      L(5)=1                                             OB1110
00117      L(8)=1                                             OB1120
00120      L(12)=1                                            OB1130
00121      L(13)=1                                            OB1140
00122      L(14)=1                                            OB1150
00123      NPL=N+L1                                           OB1160
00124      NPLP1=N+L1+1                                       OB1170
00125      NPLPG=N+L1+L2                                      OB1180
00126      L1OFF=14                                           OB1190
00127      L2OFF=L1OFF+M+N+L2                                 OB1200
00130      CRIT=0.0                                           OB1210
00131      KT=1                                               OB1211
00132      KT1=0                                              OB1212
00133      EPS=TOLP                                           OB1220
00134      IF(EPS.GT.0.0) GO TO 11                            OB1230
00136      XX=1.0E-5                                          OB1240
00137      IF(EPS.LT.0.0) XX=-EPS                             OB1250
00141      YY=0.0                                             OB1260
00142      DO 1 I=1,M                                         OB1270
00145      DO 1 J=1,M                                         OB1280
00150      EPS=A(I,J)                                         OB1290
00151      IF(EPS.LT.0.0) EPS=-EPS                            OB1300
00153    1 YY=YY+EPS                                          OB1310
00156      EPS=XX*YY/(M*N)                                    OB1311
00156 C  **STORE ORIGINAL CJ'S**                              OB1320
00157   11 DO 3 J=1,N                                         OB1330
00162    3 A(MP2,J)=A(MP1,J)                                  OB1340
00164      WRITE(6,1007)                                      OB1341
00166      DO 1008 I=1,N                                      OB1342
00171 1008 WRITE(6,1009) I,F(I)                               OB1343
00176      WRITE(6,1010) BOUND                                OB1344
00201      WRITE(6,6)                                         OB1345
00203      WRITE(6,4) KT1,CRIT                                OB1350
00207      WRITE(6,5)                                         OB1360
00207 C  **SOLVE ORIGNAL PROBLEM**                            OB1370
00211      CALL RVSMPX(A,MA,B,MB,MT,NT,L,X,EPS,INV,0)         OB1380
00212      WRITE(6,6)                                         OB1390
00214      IF(L(3).EQ.1) WRITE(6,7)                           OB1400
00217      IF(L(3).EQ.2) GO TO 910                            OB1410
00221      IF(L(3).EQ.4) GO TO 920                            OB1420
00221 C **COMPUTE C + THETA*F FOR OBJECTIVE FUNCTION VALUE**  OB1430
00223      R=0.0                                              OB1440
00224      S=0.0                                              OB1450
00225      DO 8 I=1,N                                         OB1460
00230      IF (X(I).LE.0.0) GO TO 8                           OB1470
00232      R=R+A(MP2,I)*X(I)                                  OB1480
00233      S=S+F(I)*X(I)                                      OB1490
00234    8 CONTINUE                                           OB1500
00236      ISG='+'                                            OB1510
00237      IF(R.LT.0.0) ISG=' '                               OB1520
00241      WRITE(6,9) KT1,KT,S,ISG,R                          OB1530
```

```
00250          WRITE(6,10)                                              OB1540
00250    C **COMPUTE FB*B IN ORDER OF BASIC VARIABLES**                 OB1550
00252          WRITE(6,905) (STAR,I=1,40)                               OB1551
00260      100 DO 110 I=1,M                                             OB1560
00263      110 B(MP2,I)=0.0                                             OB1570
00265          M2=0                                                     OB1580
00266          DO 130 J=1,M                                             OB1590
00271          K1=L(L2OFF+J)                                            OB1600
00272          IF(K1.GT.N) GO TO 130                                    OB1601
00274          DO 120 J1=1,M                                            OB1610
00277      120 B(MP2,J1)=B(MP2,J1)+(F(K1)*B(J,J1))                      OB1620
00301      130 CONTINUE                                                 OB1630
00301    C ***********************************************************  OB1640
00301    C **COMPUTE INDICATORS, DENOMINATORS, AND MINIMUM RATIO**      OB1650
00301    C **          FOR UNBOUNDED ORIGINAL PROBLEM                 ** OB1660
00301    C ***********************************************************  OB1670
00303          IF(L(3).NE.3) GO TO 300                                  OB1680
00305          RATMIN=1E35                                              OB1690
00306          DO 270 J=1,NPLPG                                         OB1700
00311          IF(X(J).GE.0.0) GO TO 270                                OB1710
00313          J1=L(L1OFF+J)                                            OB1720
00314          IF(J1.GT.0) GO TO 240                                    OB1730
00314    C **REAL VARIABLES**                                           OB1740
00316          YY=A(MP1,J)                                              OB1750
00317          DO 200 I=1,M                                             OB1760
00322      200 YY=YY+B(MP1,I)*A(I,J)                                    OB1770
00324          IF(YY.LE.EPS) GO TO 270                                  OB1780
00326          DO 220 I=1,M                                             OB1790
00331          XX=0.0                                                   OB1800
00332          DO 210 J1=1,M                                            OB1810
00335      210 XX=XX+B(I,J1)*A(J1,J)                                    OB1820
00337          IF(XX.GT.0.0) GO TO 270                                  OB1830
00341      220 CONTINUE                                                 OB1840
00343          XX=-F(J)                                                 OB1850
00344          DO 230 I=1,M                                             OB1860
00347      230 XX=XX+B(MP2,I)*A(I,J)                                    OB1870
00351          IF(XX.LE.0.0) GO TO 270                                  OB1880
00353          GO TO 260                                                OB1890
00353    C **SLACK AND SURPLUS VARIABLES**                              OB1900
00354      240 J2=J-N                                                   OB1910
00355          YY=(-1)**(J1+1)*B(MP1,J2)                                OB1920
00356          IF(YY.LE.EPS) GO TO 270                                  OB1930
00360          DO 250 I=1,M                                             OB1940
00363          XX=(-1)**(J1+1)*B(I,J2)                                  OB1950
00364          IF(XX.GT.0.0) GO TO 270                                  OB1960
00366      250 CONTINUE                                                 OB1970
00370          XX=(-1)**(J1+1)*B(MP2,J2)                                OB1980
00371          IF(XX.LE.0.0) GO TO 270                                  OB1990
00371    C **FIND MINIMUM RATIO**                                       OB2000
00373      260 YY=YY/XX                                                 OB2010
00374          IF(YY.GE.RATMIN) GO TO 270                               OB2020
00376          RATMIN=YY                                                OB2030
00377      270 CONTINUE                                                 OB2040
00401          IF(RATMIN.LE.1E34) GO TO 400                             OB2050
00403          WRITE(6,201)                                             OB2060
00405          WRITE(6,905) (STAR,I=1,40)                               OB2061
00413          GO TO 999                                                OB2070
00413    C ***********************************************************  OB2080
```

```
004,3   C **COMPUTE INDICATORS, DENOMINATORS, AND MINIMUM RATIO**          OB2090
004,3   C **          FOR ROUNDED ORIGINAL PROBLEM                    **   OB2100
004,3   C ********************************************************         OB2110
004,4     300 RATMIN=1E35                                                  OB2120
004,5         DO 350 J=1,NPLPG                                             OB2130
00420         IF(X(J).GE.0.0) GO TO 350                                    OB2140
00422         J1=L(L1OFF+J)                                                OB2150
00423         IF(J1.GT.0) GO TO 330                                        OB2160
00423   C **REAL VARIABLES**                                               OB2170
00425         XX=-F(J)                                                     OB2180
00426         DO 310 I=1,M                                                 OB2190
00431     310 XX=XX+B(MP2,I)*A(I,J)                                        OB2200
00433         IF(XX.GE.0.0) GO TO 350                                      OB2210
00435         YY=A(MP1,J)                                                  OB2220
00436         DO 320 I=1,M                                                 OB2230
00441     320 YY=YY+B(MP1,I)*A(I,J)                                        OB2240
00443         GO TO 340                                                    OB2250
00443   C **SLACK AND SURPLUS VARIABLES**                                  OB2260
00444     330 J2=J-N                                                       OB2270
00445         XX=(-1)**(J1+1)*B(MP2,J2)                                    OB2280
00446         IF(XX.GE.0.0) GO TO 350                                      OB2290
00450         YY=(-1)**(J1+1)*B(MP1,J2)                                    OB2300
00450   C **FIND MINIMUM RATIO**                                           OB2310
00451     340 RATIO=YY/XX                                                  OB2320
00452         IF(RATIO.GE.RATMIN) GO TO 350                                OB2330
00454         RATMIN=RATIO                                                 OB2340
00455         ENUM=YY                                                      OB2350
00456         DENOM=XX                                                     OB2360
00457         NTER=J                                                       OB2370
00460     350 CONTINUE                                                     OB2380
00462         IF(RATMIN.GT.1E34) GO TO 900                                 OB2390
00464         IF(RATMIN.GT.EPS) GO TO 400                                  OB2400
00466         M2=1                                                         OB2410
00467         RATMIN=-((1.2*EPS)-ENUM)/DENOM                               OB2420
00467   C **ACCUMULATE CRITICAL VALUE**                                    OB2430
00470     400 CRIT=CRIT+RATMIN                                             OB2440
00471         IF(L(3).EQ.3) WRITE(6,401) CRIT                              OB2450
00475         IF(CRIT.LE.ROUND) GO TO 500                                  OB2460
00477         WRITE(6,402) ROUND                                           OB2470
00502         ISG='+'                                                      OB2480
00503         IF(R.LT.0.0) ISG=' '                                         OB2490
00505         KT1=KT                                                       OB2500
00506         KT=KT+1                                                      OB2501
00507         WRITE(6,9) KT1,KT,S,ISG,R                                    OB2502
005,6         WRITE(6,905) (STAR,I=1,40)                                   OB2503
005,4         GO TO 999                                                    OB2510
00524   C **********************************************************       OB2520
00524   C **UPDATE OBJECTIVE FUNCTION WITH NEW THETA AND F VECTOR**        OB2530
00524   C **********************************************************       OB2540
00525     500 DO 510 J=1,N                                                 OB2550
00530         A(MP1,J)=A(MP1,J)+(RATMIN*F(J))                              OB2560
00531     510 CONTINUE                                                     OB2570
00533         IF(L(3).EQ.3.OR.M2.EQ.1) GO TO 800                           OB2580
00533   C ******************************************************           OB2590
00533   C **GET APPROPRIATE Y(J) AND STORE IN COLUMN MP1 OF B**            OB2600
00533   C ******************************************************           OB2610
00535         K3=L(L1OFF+NTER)                                             OB2620
00536         IF(K3.EQ.0) GO TO 600                                        OB2630
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```
00540            IF(K3.EQ.1.OR.K3.EQ.2) GO TO 620                      OB2640
00542        600 DO 610 I=1,M                                          OB2650
00545            B(I,MP1)=0.0                                          OB2660
00546            DO 610 J1=1,M                                         OB2670
00551        610 B(I,MP1)=B(I,MP1)+B(I,J1)*A(J1,NTER)                  OB2680
00554            GO TO 700                                             OB2690
00555        620 J2=NTER-N                                             OB2700
00556            DO 630 I=1,M                                          OB2710
00561        630 B(I,MP1)=(-1)**(K3+1)*B(I,J2)                         OB2720
00561      C *******************************************               OB2730
00561      C **FIND THE LEAVING AND ENTERING VARIABLES**               OB2740
00561      C *******************************************               OB2750
00563        700 SMALL=1E35                                            OB2760
00564            J5=0                                                  OB2770
00565            DO 710 J=1,M                                          OB2780
00570            IF(B(J,MP1).LE.0.0) GO TO 710                         OB2790
00572            J5=1                                                  OB2800
00573            TEST=B(J,MP2)/B(J,MP1)                                OB2810
00574            IF(TEST.GT.SMALL) GO TO 710                           OB2820
00576            SMALL=TEST                                            OB2830
00577            LEAVE=L(L2OFF+J)                                      OB2840
00600        710 CONTINUE                                              OB2850
00602            KT1=KT                                                OB2860
00603            KT=KT+1                                               OB2861
00604            IF(NSET.EQ.1.AND.J5.EQ.0) WRITE(6,5)                  OB2862
00607            IF(NSET.EQ.1.AND.J5.EQ.0) WRITE(6,1105)KT1,CRIT       OB2863
00614            IF(NSET.EQ.1.AND.J5.EQ.0) WRITE(6,905) (STAR,I=1,40)  OB2864
00623            IF(NSET.EQ.1.AND.J5.EQ.0) GO TO 999                   OB2865
00625            IF(NSET.EQ.1) GO TO 1001                              OB2866
00627            WRITE(6,4) KT1,CRIT                                   OB2867
00633            IF(J5.EQ.0) GO TO 930                                 OB2870
00633      C **SET X VECTOR FOR RVSMPX WITH ADVANCED START**           OB2880
00635       1001 X(LEAVE)=-1.0                                         OB2890
00636            X(NTER)=1.0                                           OB2900
00637            IF(NTER.GT.N) GO TO 720                               OB2910
00641            LET1=' X('                                            OB2920
00642            GO TO 730                                             OB2930
00643        720 LET1=' S('                                            OB2940
00644            NTER=NTER-N                                           OB2950
00645        730 IF(LEAVE.GT.N) GO TO 740                              OB2960
00647            LET2=' X('                                            OB2970
00650            GO TO 750                                             OB2980
00651        740 LET2=' S('                                            OB2990
00652            LEAVE=LEAVE-N                                         OB3000
00653            IF(NSET.EQ.1) L(13)=0                                 OB3002
00655        750 IF(NSET.EQ.1) GO TO 1002                              OB3004
00657            WRITE(6,751) LET1,NTER,LET2,LEAVE                     OB3010
00665       1002 L(1)=L1                                               OB3020
00666            L(2)=L2                                               OB3030
00667            L(3)=0                                                OB3040
00670            L(4)=0                                                OB3050
00671            L(12)=0                                               OB3051
00672            INV=1                                                 OB3060
00673            CALL RVSMPX(A,MA,B,MB,MT,NT,L,X,EPS,INV,0)            OB3070
00674            WRITE(6,6)                                            OB3080
00676            IF(NSET.EQ.1) GO TO 1003                              OB3082
00700            WRITE(6,906) X(M+N+L2+1)                              OB3084
00703       1003 R=0.0                                                 OB3090
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00704        S=0.0                                                        OB3100
00705        DO 760 I=1,N                                                 OP3110
00710        IF(X(I).LE.0.0) GO TO 760                                    OB3120
00712        R=R+A(MP2,I)*X(I)                                            OB3130
00713        S=S+F(I)*X(I)                                                OB3140
00714    760 CONTINUE                                                     OB3150
00716        ISG='+'                                                      OB3160
00717        IF(R.LT.0.0) ISG=' '                                         OB3170
00721        IF(NSET.EQ.1.AND.KT1.EQ.1) WRITE(6,1004)                     OB3172
00724        IF(NSET.EQ.1.AND.KT1.EQ.1) WRITE(6,1005)                     OB3174
00727        IF(NSET.EQ.1.AND.KT1.EQ.1) WRITE(6,5)                        OB3176
00732        IF(NSET.EQ.1) WRITE(6,1006)KT1,CRIT,LET1,NTFR,LET2,LEAVE,X(M+N+L2+  OB3177
00732       11),S,ISG,R                                                   OB3178
00747        IF(NSET.EQ.1) GO TO 100                                      OB3179
00751        WRITE(6,9) KT1,KT,S,ISG,R                                    OB3180
00760        WRITE(6,905) (STAR,I=1,40)                                   OB3190
00766        GO TO 100                                                    OB3200
00767    800 KT1=KT                                                       OB3210
00770        KT=KT+1                                                      OB3211
00771        IF(NSET.EQ.1) GO TO 1099                                     OB3212
00773        WRITE(6,4) KT1,CRIT                                          OB3213
00777   1099 L(1)=L1                                                      OB3220
01000        L(2)=L2                                                      OP3230
01001        IF(NSET.EQ.1) GO TO 1100                                     OB3231
01003        L(3)=L3                                                      OB3240
01004   1100 INV=1                                                        OB3242
01005        IF(NSET.NE.1) GO TO 1101                                     OB3244
01007        L(3)=0                                                       OB3246
01010        L(4)=0                                                       OB3248
01011        L(12)=0                                                      OB3250
01012        L(13)=0                                                      OB3252
01013   1101 CALL RVSMPX(A,MA,B,MB,MT,NT,L,X,EPS,INV,0)                  OB3260
01014        WRITE(6,5)                                                   OB3270
01016        R=0.0                                                        OB3280
01017        S=0.0                                                        OB3290
01020        DO 810 I=1,N                                                 OB3300
01023        IF(X(I).LE.0.0) GO TO 810                                    OB3310
01025        R=R+A(MP2,I)*X(I)                                            OB3320
01026        S=S+F(I)*X(I)                                                OB3330
01027    810 CONTINUE                                                     OB3340
01031        ISG='+'                                                      OB3350
01032        IF(R.LT.0.0) ISG=' '                                         OB3360
01034        IF(NSET.EQ.1.AND.KT1.EQ.1) WRITE(6,1004)                     OB3361
01037        IF(NSET.EQ.1.AND.KT1.EQ.1) WRITE(6,1005)                     OB3362
01042        IF(NSET.EQ.1.AND.KT1.EQ.1) WRITE(6,5)                        OB3363
01045        IF(NSET.EQ.1) WRITE(6,1102) KT1,CRIT,X(M+N+L2+1),S,ISG,R     OB3364
01056        IF(NSET.EQ.1) WRITE(6,1103)                                  OB3365
01061        IF(NSET.EQ.1) WRITE(6,1104)                                  OB3366
01064        IF(NSET.EQ.1) GO TO 100                                      OB3367
01066        WRITE(6,9) KT1,KT,S,ISG,R                                    OB3370
01075        WRITE(6,10)                                                  OB3380
01077        WRITE(6,905) (STAR,I=1,40)                                   OB3381
01105        GO TO 100                                                    OB3390
01106    900 WRITE(6,901)                                                 OB3400
01110        GO TO 999                                                    OB3410
01111    910 WRITE(6,902)                                                 OB3420
01113        GO TO 999                                                    OB3430
01114    920 WRITE(6,903)                                                 OB3440
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01116            GO TO 999                                                        OB3450
01117        930 WRITE(6,904)                                                     OB3460
01121            WRITE(6,905) (STAR,I=1,40)                                       OB3461
01127        999 DO 1000 J=1,N                                                    OB3470
01132            A(MP1,J)=A(MP2,J)                                                OB3480
01133       1000 CONTINUE                                                         OB3490
01135          4 FORMAT(1H0,'CRITICAL VALUE (',I3,') OF THETA = ',E14.8)          OB3500
01136          5 FORMAT(//)                                                       OB3510
01137          6 FORMAT(/)                                                        OB3520
01140          7 FORMAT(' NUMERICAL DIFFICULTIES IN RVSMPX.  QUESTION RESULTS OF PA  OB3530
01140           1RAMETRIC STUDY')                                                 OB3540
01141          9 FORMAT(' OBJECTIVE FUNCTION VALUE BETWEEN CRITICAL VALUES (',I3,')  OB3550
01141           1 AND (',I3,') OBTAINED FROM ',E15.8,'* THETA 'A1,E15.8)          OB3560
01142         10 FORMAT(///)                                                      OB3570
01143        201 FORMAT(' PROBLEM REMAINS UNBOUNDED THROUGHOUT RANGE OF CRITICAL VA  OB3580
01143           1LUES')                                                           OB3590
01144        401 FORMAT(' WHEN THETA =',F15.10,'YOUR PROBLEM BECOMES BOUNDED AND PR  OB3600
01144           1OCEEDS')                                                         OB3610
01145        402 FORMAT(' CRITICAL VALUE OF THETA BYPASSES YOUR LIMITATION (BOUND),  OB3620
01145           1 THEREFORE THIS BASIS REMAINS OPTIMAL AT YOUR LIMIT OF ',F10.4)  OB3630
01146        751 FORMAT(' AT THAT VALUE,'A3,I4,') ENTERS THE BASIS AND'A3,I4,') LEA  OB3640
01146           1VES.')                                                           OB3650
01147        901 FORMAT(' THE CRITICAL VALUE OF THETA IS INFINITE.')              OB3660
01150        902 FORMAT(' PROBLEM IS INFEASIBLE.  NO STUDY PERFORMED')            OB3670
01151        903 FORMAT(' SYSTEM ERROR ENCOUNTERED BY RVSMPX.  NO STUDY PERFORMED')  OB3680
01152        904 FORMAT(' PROBLEM GOES UNBOUNDED BEYOND THIS CRITICAL VALUE.')    OB3690
01153        905 FORMAT(/40A3)                                                    OB3691
01154        906 FORMAT(' OBJECTIVE FUNCTION VALUE =', E14.8)                     OB3692
01155       1004 FORMAT(' CRIT VAL     CRIT VAL     BASIS INFORMATION     OBJ FUNC   OB3693
01155           1VAL     OBJ FUNC VAL BETWEEN THIS')                              OB3694
01156       1005 FORMAT('   NO.                          ENTERS     LEAVES',15X,'  OB3695
01156           1    AND NEXT CRIT VAL')                                          OB3696
01157       1006 FORMAT(I6,F15.4,5X,A3,I4,')',3X,A3,I4,')',F14.4,6X,F10.4,'* THETA  OB3697
01157           1',A1,F10.4)                                                      OB3698
01160       1007 FORMAT(' THE F VECTOR IS AS FOLLOWS')                            OB3700
01161       1009 FORMAT(' F(',I4,') =',F6.2)                                      OB3710
01162       1010 FORMAT(' BOUND=',F6.2)                                           OB3720
01163       1102 FORMAT(I6,F15.4,5X,' NO REPORT - THETA',F14.4,6X,F10.4,'* THETA ',  OB3730
01163           1A1,F10.4)                                                        OB3740
01164       1103 FORMAT(26X,' LESS THAN EPS OR ')                                 OB3750
01165       1104 FORMAT(26X,' ORIG PROB UNBOUNDED')                               OB3760
01166       1105 FORMAT(I6,F15.4,5X,' NO FURTHER INFORMATION-PROBLEM GOES UNBOUNDED
01166           1 BEYOND THIS CRITICAL VALUE')
01167            MT=MT+1                                                          OB3770
01170            RETURN                                                           OB3780
01171            END                                                              OB3790

      END OF COMPILATION:        NO  DIAGNOSTICS.


@XQT
MAP 0023-09/20-11:43
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APPENDIX C: Listing of PARRHS

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@FOR,IS PARRHS
FOR C10A-09/21/73-19:31:09 (,0)

   SUBROUTINE PARRHS     ENTRY POINT 002027

   STORAGE USED: CODE(1) 002233; DATA(0) 000447; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003    RVSMPX
    0004    NWDU$
    0005    NI02$
    0006    NI01$
    0007    NEXP1$
    0010    NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0000    000060 10F         0000    000276 1001F       0000    000304 1003F       0000    000310 1004F       0000    000313 1005F
    0001    001063 1006L       0001    001206 1007L       0001    001556 1008L       0000    000330 1009F       0000    000351 1010F
    0001    000141 133G        0000    000101 14F         0001    000155 140G        0001    001716 15L         0001    000174 151G
    0000    000103 16F         0000    000121 17F         0000    000132 18F         0000    000144 19F         0001    000622 199L
    0001    000626 200L        0001    000323 206G        0001    000345 217G        0001    000354 224G        0001    000726 230L
    0000    000164 231F        0001    000441 254G        0001    000524 266G        0001    000547 277G        0000    000045 3F
    0001    001130 300L        0000    000212 301F        0001    001676 302L        0001    000557 304G        0001    001173 310L
    0001    000615 320G        0001    000664 330G        0001    000670 334G        0001    000707 345G        0000    000056 4F
    0001    001057 414G        0001    001106 423G        0001    001166 453G        0001    001243 470G        0000    000057 5F
    0001    001273 501G        0001    001314 511G        0001    001427 540G        0001    001322 540L        0001    001454 546G
    0001    001370 550L        0001    001403 560L        0000    000236 561F        0001    001434 570L        0001    001711 642G
    0001    001745 657G        0001    001772 664G        0001    000446 8L          0001    000454 9L          0001    001505 900L
    0001    001512 910L        0001    001522 920L        0001    001527 930L        0000    000253 931F        0000    000251 950F
    0000    000267 951F        0001    001751 999L        0000 R  000026 CBY         0000 R  000030 COMPUT      0000 R  000007 CRIT
    0000 I  000021 I           0000    000403 INJP$       0000 I  000031 ISG         0000 I  000022 J           0000 I  000041 JI
    0000 I  000044 J2          0000 I  000027 K           0000 I  000035 KLEAVE      0000 I  000023 KT          0000 I  000024 KTI
    0000 I  000037 LEAVE       0000 I  000032 LETI        0000 I  000034 LET2        0000 I  000010 LI          0000 I  000017 LIOFF
    0000 I  000011 L2          0000 I  000020 L2OFF       0000 I  000012 L3          0000 I  000000 M           0000 I  000003 MPI
    0000 I  000002 MP2         0000 I  000001 N           0000 I  000016 NMGI        0000 I  000013 NPL         0000 I  000015 NPLPG
    0000 I  000014 NPLPI       0000 I  000004 NPI         0000 I  000005 NP2         0000 I  000033 NTER        0000 R  000036 RATIO
    0000 R  000040 RATMAX      0000 R  000025 RATMIN      0000 R  000006 STAR        0000 R  000042 XX          0000 R  000043 YY


00101       1*          SUBROUTINE PARRHS(A,MA,B,MB,MT,NT,L,X,TOLP,INV,BOUND,R,NSET)      RH1000
00103       2*          DIMENSION A(MA,1),B(MB,1),L(1),X(1),R(1)                          RH1010
00104       3*          M=MT-2                                                            RH1020
00105       4*          N=NT-2                                                            RH1030
00106       5*          MP2=M+2                                                           RH1040
00107       6*          MPI=M+1                                                           RH1050
00110       7*          NPI=N+1                                                           RH1060
00111       8*          NP2=N+2                                                           RH1070
00112       9*          STAR='  *'                                                        RH1071
00113      10*          CRIT=0.0                                                          RH1080
00114      11*          LI=L(1)                                                           RH1090
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00115          L2=L(2)                                                  RH1100
00116          L3=L(3)                                                  RH1110
00117          L(5)=1                                                   RH1120
00120          L(8)=1                                                   RH1130
00121          L(12)=1                                                  RH1140
00122          L(13)=1                                                  RH1150
00123          L(14)=1                                                  RH1160
00124          NPL=N+11                                                 RH1170
00125          NPLP1=NPL+1                                              RH1180
00126          NPLPG=NPL+L2                                             RH1190
00127          NMG1=N+M+L2+1                                            RH1200
00130          L1OFF=14                                                 RH1210
00131          L2OFF=L1OFF+M+N+L2                                       RH1220
00132          DO 1 I=1,M                                               RH1230
00135        1 A(I,NP2)=A(I,NP1)                                        RH1240
00137          DO 2 J=1,NP1                                             RH1250
00142        2 A(MP2,J)=A(MP1,J)                                        RH1260
00144          KT=1                                                     RH1270
00145          KT1=0                                                    RH1271
00146          WRITE(6,1001)                                            RH1272
00150          DO 1002 I=1,M                                            RH1273
00153     1002 WRITE(6,1003) I,R(I)                                     RH1274
00160          WRITE(6,5)                                               RH1275
00162          WRITE(6,1004) BOUND                                      RH1276
00165          WRITE(6,5)                                               RH1277
00167          WRITE(6,3) KT1,CRIT                                      RH1278
00173          WRITE(6,4)                                               RH1280
00173     C *************************                                   RH1290
00173     C **SOLVE ORIGINAL PROBLEM**                                  RH1300
00173     C *************************                                   RH1310
00175          MT=MT-1                                                  RH1320
00176          NT=NT-1                                                  RH1330
00177          CALL RVSMPX(A,MA,B,MB,MT,NT,L,X,TOLP,INV,0)              RH1340
00200          WRITE(6,5)                                               RH1350
00202          IF(L(3).LT.2) GO TO 200                                  RH1360
00204          WRITE(6,950) (STAR,I=1,40)                               RH1361
00212          IF(L(3).EQ.3) GO TO 15                                   RH1370
00212     C **INFEASIBILITY METHOD FOR INFEASIBLE ORIGINAL PROBLEM**    RH1380
00214          NT=NT+1                                                  RH1390
00215          A(MP1,NP1) =-1.0                                         RH1400
00216          DO 6 J=1,N                                               RH1410
00221        6 A(MP1,J)=0.0                                             RH1420
00223          DO 7 I=1,M                                               RH1430
00226          A(I,NP1)=-R(I)                                           RH1440
00227        7 CONTINUE                                                 RH1450
00231          L(1)=L1                                                  RH1460
00232          L(2)=L2                                                  RH1470
00233          L(3)=L3                                                  RH1480
00234          INV=0                                                    RH1490
00235          CALL RVSMPX(A,MA,B,MB,MT,NT,L,X,TOLP,INV,0)              RH1500
00236          WRITE(6,14)                                              RH1510
00240          NT=NT-1                                                  RH1520
00241          IF(L(3).EQ.2) GO TO 9                                    RH1530
00243          RATMIN=X(NP1)                                            RH1540
00244          CRIT=RATMIN                                              RH1550
00245          IF(CRIT.LE.BOUND) GO TO 8                                RH1560
00247          WRITE(6,10) CRIT                                         RH1570
00252          WRITE(6,950) (STAR,I=1,40)                               RH1571
```

```
00260        GO TO 9                                                           RH1580
00261      8 WRITE(6,19) CRIT                                                  RH1590
00264      9 A(MP1,NP1)=A(MP2,NP1)                                             RH1600
00265        DO 11 J=1,M                                                       RH1610
00270     11 A(MP1,J)=A(MP2,J)                                                 RH1620
00272        IF(L(3).EQ.2) GO TO 15                                            RH1630
00274        IF(CRIT.GT.BOUND) GO TO 999                                       RH1640
00276        DO 12 I=1,M                                                       RH1650
00301     12 A(I,NP1)=A(I,NP2)                                                 RH1660
00303        DO 13 I=1,M                                                       RH1670
00306        A(I,NP1)=A(I,NP1)+RATMIN*R(I)                                     RH1680
00307     13 CONTINUE                                                          RH1690
00311        L(1)=L1                                                           RH1700
00312        L(2)=L2                                                           RH1710
00313        L(3)=L3                                                           RH1720
00314        INV=0                                                             RH1730
00314 C **SOLVE PROBLEM WITH THETA FROM INFEASIBILITY METHOD**                   RH1740
00315        CALL RVSMPX(A,MA,B,MB,MT,NT,L,X,TOLP,INV,0)                       RH1750
00316        WRITE(6,950) (STAR,I=1,40)                                        RH1760
00316 C ********************************************                           RH1770
00316 C **COMPUTE Y AND STORE IN COLUMN MP1 OF B**                              RH1780
00316 C ********************************************                           RH1790
00324        GO TO 200                                                         RH1791
00325    199 KT1=KT                                                            RH1792
00326        KT=KT+1                                                           RH1793
00327    200 DO 220 I=1,M                                                      RH1800
00332        B(I,MP1)=0.0                                                      RH1810
00333        DO 210 J=1,M                                                      RH1820
00336        B(I,MP1)=B(I,MP1)+(B(I,J)*R(J))                                   RH1830
00337    210 CONTINUE                                                          RH1840
00341    220 CONTINUE                                                          RH1850
00343        CBY=0.0                                                           RH1860
00344        DO 230 J=1,M                                                      RH1870
00347        K=L(L2OFF+J)                                                      RH1880
00350        IF(K.GT.N) GO TO 230                                              RH1890
00352        CBY=CBY+(A(MP2,K)*B(J,MP1))                                       RH1900
00353    230 CONTINUE                                                          RH1910
00355        COMPUT=X(NMG1)-(CBY*CRIT)                                         RH1920
00356        ISG='+'                                                           RH1930
00357        IF(COMPUT.LT.0.0) ISG=' '                                         RH1940
00361        IF(NSET.EQ.1.AND.KT1.GT.0) WRITE(6,1005)KT1,CRIT,LET1,NTER,LET2,KL RH1942
00361       1EAVE,X(NMG1),CBY,ISG,COMPUT                                       RH1944
00376        IF(NSET.EQ.1.AND.KT1.GT.0) WRITE(6,4)                             RH1946
00401        IF(NSET.EQ.1.AND.KT1.GT.0) GO TO 1006                             RH1948
00403        WRITE(6,231) KT1,KT,CBY,ISG,COMPUT                                RH1950
00412        WRITE(6,950) (STAR,I=1,40)                                        RH1960
00412 C **FIND MINIMUM RATIO AND LEAVING VARIABLE**                             RH1970
00420   1006 K=0                                                               RH1980
00421        RATMIN=1E35                                                       RH1990
00422        DO 300 I=1,M                                                      RH2000
00425        IF(B(I,MP1).GE.0.0) GO TO 300                                     RH2010
00427        RATIO=-B(I,MP2)/B(I,MP1)                                          RH2020
00430        IF(RATIO.GT.RATMIN) GO TO 300                                     RH2030
00432        RATMIN=RATIO                                                      RH2040
00433        K=I                                                               RH2050
00434        LEAVE=I                                                           RH2060
00435    300 CONTINUE                                                          RH2070
00437        KLEAVE=L(L2OFF+LEAVE)                                             RH2080
```

```
00440   128.        CRIT=CRIT+RATMIN                                                RH2090
00441   129.        IF(K.EQ.0) GO TO 302                                            RH2100
00443   130.        IF(CRIT.LE.ROUND) GO TO 310                                     RH2110
00445   131.        WRITE(6,301) KT,ROUND                                           RH2120
00451   132.        WRITE(6,950) (STAR,I=1,40)                                      RH2121
00457   133.        GO TO 999                                                       RH2130
00460   134.    310 IF(NSET.EQ.1) GO TO 1007                                        RH2135
00462   135.        WRITE(6,3) KT,CRIT                                              RH2140
00462   136.  C ****************************************************************    RH2150
00462   137.  C **COMPUTE INDICATORS AND DENOMINATORS FOR ENTERING VARIABLES**      RH2160
00462   138.  C ****************************************************************    RH2170
00466   139.   1007 RATMAX=-1E35                                                    RH2180
00467   140.        DO 560 J=1,NPLPG                                                RH2190
00472   141.        IF(X(J).GE.0.0) GO TO 560                                       RH2200
00474   142.        J1=L(LOFF+J)                                                    RH2210
00475   143.        IF(J1.GT.0) GO TO 540                                           RH2220
00475   144.  C **REAL VARIABLES**                                                  RH2230
00477   145.        XX=0.0                                                          RH2240
00500   146.        DO 510 I=1,M                                                    RH2250
00503   147.    510 XX=XX+B(LEAVE,I)*A(I,J)                                         RH2260
00505   148.        IF(XX.GE.0.0) GO TO 560                                         RH2270
00507   149.        YY=A(MP1,J)                                                     RH2280
00510   150.        DO 520 I=1,M                                                    RH2290
00513   151.    520 YY=YY+B(MP1,I)*A(I,J)                                           RH2300
00515   152.        GO TO 550                                                       RH2310
00515   153.  C **SLACK AND SURPLUS VARIABLES**                                     RH2320
00516   154.    540 J2=J-N                                                          RH2330
00517   155.        XX=(-1)**(J1+1)*B(LEAVE,J2)                                     RH2340
00520   156.        IF(XX.GE.0.0) GO TO 560                                         RH2350
00522   157.        YY=(-1)**(J1+1)*B(MP1,J2)                                       RH2360
00523   158.    550 RATIO=-YY/XX                                                    RH2370
00524   159.        IF(RATIO.LE.RATMAX) GO TO 560                                   RH2380
00526   160.        RATMAX=RATIO                                                    RH2390
00527   161.        NTER=J                                                          RH2400
00530   162.    560 CONTINUE                                                        RH2410
00532   163.        IF(RATMAX.GT.-1E35) GO TO 570                                   RH2420
00534   164.        WRITE(6,561)                                                    RH2430
00536   165.        WRITE(6,950) (STAR,I=1,40)                                      RH2431
00544   166.        GO TO 999                                                       RH2440
00544   167.  C **************************                                          RH2450
00544   168.  C **CHANGE RIGHT HAND SIDES**                                         RH2460
00544   169.  C **************************                                          RH2470
00545   170.    570 DO 580 I=1,M                                                    RH2480
00550   171.        A(I,NP1)=A(I,NP1)+(RATMIN*R(I))                                 RH2490
00551   172.    580 CONTINUE                                                        RH2500
00551   173.  C **SET X VECTOR FOR RVSMPX WITH ADVANCED START**                     RH2510
00553   174.        X(NTER)=1.0                                                     RH2520
00554   175.        X(KLEAVE)=-1.0                                                  RH2530
00555   176.        L(1)=L1                                                         RH2540
00556   177.        L(2)=L2                                                         RH2550
00557   178.        L(3)=0                                                          RH2560
00560   179.        L(4)=0                                                          RH2570
00561   180.        L(12)=0                                                         RH2571
00562   181.        INV=1                                                           RH2580
00563   182.        IF(NTER.GT.N) GO TO 900                                         RH2590
00565   183.        LETI=' X('                                                      RH2600
00566   184.        GO TO 910                                                       RH2610
00567   185.    900 LETI=' S('                                                      RH2620
```

```
      NTER=NTER-N                                                       RH2630
  910 IF(KLEAVE.GT.N) GO TO 920                                         RH2640
      LET2=' X('                                                        RH2650
      GO TO 930                                                         RH2660
  920 LET2=' S('                                                        RH2670
      KLEAVE=KLEAVE-N                                                   RH2680
  930 IF(NSET.EQ.1) L(13)=0                                             RH2682
      IF(NSET.EQ.1) GO TO 1008                                          RH2684
      WRITE(6,931) LET1,NTER,LET2,KLEAVE                                RH2690
      WRITE(6,5)                                                        RH2691
 1008 CALL RVSMPX(A,MA,B,MB,MT,NT,L,X,TOLP,INV,0)                       RH2700
      WRITE(6,5)                                                        RH2710
      IF(NSET.EQ.1.AND.KT1.EQ.0) WRITE(6,1009)                          RH2712
      IF(NSET.EQ.1.AND.KT1.EQ.0) WRITE(6,1010)                          RH2714
      IF(NSET.EQ.1.AND.KT1.EQ.0) WRITE(6,4)                             RH2716
      IF(NSET.EQ.1) GO TO 199                                           RH2718
      WRITE(6,951) X(NMG1)                                              RH2720
      GO TO 199                                                         RH2721
  302 WRITE(6,18) KT                                                    RH2730
      WRITE(6,950) (STAR,I=1,40)                                        RH2731
      GO TO 999                                                         RH2732
   15 IF(L(3).EQ.2) WRITE(6,16)                                         RH2740
      IF(L(3).EQ.3) WRITE(6,17)                                         RH2750
      WRITE(6,950) (STAR,I=1,40)                                        RH2751
  999 DO 1000 I=1,M                                                     RH2760
 1000 A(I,NP1)=A(I,NP2)                                                 RH2770
    3 FORMAT(1H0,'CRITICAL VALUE (',I3,') OF THETA = ',E14.8)           RH2780
    4 FORMAT(//)                                                        RH2790
    5 FORMAT(/)                                                         RH2800
   10 FORMAT(' WHEN THETA CRITICAL =', F15.5, 'YOUR PROBLEM BECOMES FEAS RH2810
     1IBLE BUT THIS BYPASSES YOUR LIMIT.')                              RH2820
   14 FORMAT(/////)                                                     RH2830
   16 FORMAT(' PROBLEM REMAINS INFEASIBLE THROUGHOUT RANGE OF THETA.  NO RH2840
     1 STUDY PERFORMED.')                                               RH2850
   17 FORMAT(' PROBLEM IS UNBOUNDED.  NO STUDY PERFORMED.')             RH2860
   18 FORMAT('      CRITICAL VALUE (',I3,') OF THETA IS INFINITE.')     RH2870
   19 FORMAT(' WHEN THETA CRITICAL =', F15.5, 'YOUR PROBLEM BECOMES FEAS RH2880
     1IBLE AND THE STUDY CONTINUES.')                                   RH2890
  231 FORMAT(' OBJECTIVE FUNCTION VALUE BETWEEN CRITICAL VALUES (',I3,') RH2900
     1 AND (',I3,') OF THETA OBTAINED FROM',E15.8,'*THETA ',A1,E15.8)   RH2910
  301 FORMAT('      CRITICAL VALUE (',I3,') BYPASSES YOUR LIMITATION, THE RH2920
     1REFORE BASIS REMAINS OPTIMAL AT YOUR LIMIT OF',F10.5)             RH2930
  561 FORMAT(' PROBLEM BECOMES INFEASIBLE BEYOND THIS CRITICAL VALUE.') RH2940
  950 FORMAT(/40A3)                                                     RH2941
  931 FORMAT(' AT THAT VALUE,'A3,I4,') ENTERS THE BASIS AND'A3,I4,') LEA RH2950
     1VES.')                                                            RH2960
  951 FORMAT(' OBJECTIVE FUNCTION VALUE =', E14.8)                      RH2961
 1001 FORMAT(' THE R VECTOR IS AS FOLLOWS')                             RH2962
 1003 FORMAT(' R('I4,') ='F6.2)                                         RH2964
 1004 FORMAT(' BOUND='F6.2)                                             RH2966
 1005 FORMAT(I6,F15.4,5X,A3,I4,')',3X,A3,I4,')',F14.4,6X,F10.4,'* THETA RH2968
     1',A1,F10.4)                                                       RH2970
 1009 FORMAT(' CRIT VAL      CRIT VAL      BASIS INFORMATION     OBJ FUNC RH2972
     1VAL      OBJ FUNC VAL BETWEEN THIS')                              RH2974
 1010 FORMAT('   NO.                        ENTERS     LEAVES',15X,'    RH2976
     1   AND NEXT CRIT VAL')                                            RH2978
      MT=MT+1                                                           RH2980
      NT=NT+1                                                           RH2982
```

APPENDIX D: Timing Considerations

APPENDIX D

TIMING CONSIDERATIONS

RANGES SAMPLES: Time Per Right-Hand Side

and Objective Function Coefficient

PROBLEM SIZE		ELAPSED TIME RANGES (SEC.)	TIME PER R.H.S.	TIME PER COEFFICIENT
m	n			
10	20	.1388	.0031	.0054
15	20	.1674	.0031	.0060
20	20	.2216	.0035	.0075
20	30	.2750	.0032	.0071
20	80	.9258	.0033	.0108
50	150	4.0446	.0039	.0257

APPENDIX D (continued)

PAROBJ SAMPLES: Time Per Critical Value

PROBLEM SIZE		ELAPSED TIME PAROBJ (SEC.)	NO. OF COMPUTED CRITICAL VALUES	TIME PER CRITICAL VALUE
m	n			
10	20	.7608	3	.2536
15	20	.0554	1	.0554
20	20	14.1665	5	2.8333
20	30	12.5100	9	1.3900
20	80	9.4508	8	1.1814
50	150	190.2708	23	8.2726

APPENDIX D (continued)

PARRHS SAMPLES: Time Per Critical Value

PROBLEM SIZE m	n	ELAPSED TIME PARRHS (SEC.)	NO. OF COMPUTED CRITICAL VALUES	TIME PER CRITICAL VALUE
10	20	.7930	2	.3965
15	20	1.8704	3	.6235
20	20	4.3180	10	.3925
20	30	1.9914	1	1.9914
20	80	4.7322	1	4.7322
50	150	73.2580	2	36.6290

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16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

This report is a sequel to NBS Report 10695 (February 1972), "The National Bureau of Standards' Linear and Quadratic Programming Subroutines," which documented one phase of an effort to provide users, of the facility operated by the National Bureau of Standards' Computer Services Division, with reliable, clearly-described solution algorithms for selected frequently-arising classes of special mathematical problems. The present report presents subroutines which perform post-optimality analysis and parametric programming studies on linear programming problems solved by the National Bureau of Standards' RVSMPX subroutine. (The present versions of these codes use internal storage only.)

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